Image Segmentation via Normalised Cuts and Clustering Algorithm

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Abstract—**Image segmentation has been widely applied in image analysis for various areas such as biomedical imaging, intelligent transportation systems and satellite imaging. The main goal of image segmentation is to simplify an image into segments that have a strong correlation with objects in the real world. Homogeneous regions of an image are regions containing common characteristics and are grouped as single segment. One of the graph partitioning methods in image segmentation, normalised cuts, has been recognised producing reliable segmentation result. To date, normalised cuts in image segmentation of various sized images is still lacking of analysis of its performance. In this paper, segmentation on synthetic images and natural images are covered to study the performance and effect of different image complexity towards segmentation process. This study gives some research findings for effective image segmentation using graph partitioning method with computation cost reduced. Because of its cost expensive and it becomes unfavourable in performing image segmentation on high resolution image especially in online image retrieval systems. Thus, a graph-based image segmentation method done in multistage approach is introduced here.**

Index Terms—**Image segmentation; normalised cuts; graph partitioning.**

I. INTRODUCTION

Image segmentation can be generally regarded as partitioning an image into multiple segments. The segmentation process provides a more simplified image representation as these segments can be individually analysed without the need of human to do manual segmentation at first hand [1]. There are vast variety of segmentation methods such as simple straight forward segmentation with just determining the foreground and background of the image. This basic segmentation is not sufficient in meeting demands from the current trend of image representation especially in object recognition application. A more reliable segmentation is needed to counter more complicated cases by applying some useful domains. Colour information is one of the popular domains used for image segmentation.

To date, there are few approaches of image segmentation methods can be categorised: global knowledge based segmentation, edge-based segmentation and region based segmentation [2]. The global knowledge based segmentation identifies the threshold from a greyscale or colour intensity histogram representing an image. This threshold acts as splitting boundary to segment the image into foreground and background regions. This thresholding method is primitive such that obtaining the optimal threshold may not be easy when the image is low in contrast and contains multiple thresholds. Edge based segmentation is more likely suitable for line seeking application such as text recognition. Discontinuity features are sought by using some of the popular edge detectors: Canny edge detector, Prewitt and Sobel operators [3]. Edge detector alone may not be good enough to do the segmentation since it does not guarantee forming closed boundaries which is important for recognising distinguishable segments [4]. It normally serves as base of a segmentation method. One of the region based method, graph partitioning, is popular for complex image segmentation especially natural image with tiny objects. Shi and Malik had formulated an improved graph partitioning algorithm called normalised cuts [5]. Graph partitioning does segmentation by viewing an image as a graph. Due to its algorithm complexity and finding the minimum cut for the segmentation is a NP-complete problem, it is unsuitable to be applied in real time system [1]. Hence, improving normalised cuts algorithm in image segmentation is presented in this paper. The structure of this paper is as followings. In section II, the background of graph theoretic formulation terms is described. Section III describes the normalised cuts implementation in image segmentation. The proposed method for improving normalised cuts algorithm is described in section IV. Experimental result of the image segmentation is presented in section V. Lastly, this paper is concluded in section VI.

II. GRAPH PARTITIONING

A. Vertices and Edges

A graph $G = (V, E)$, is constructed with collection of vertices, V and edges, E. This graph theory is generally used in modelling problems such as traffic networks, electrical circuits and internet networks [6]. To form graph in image

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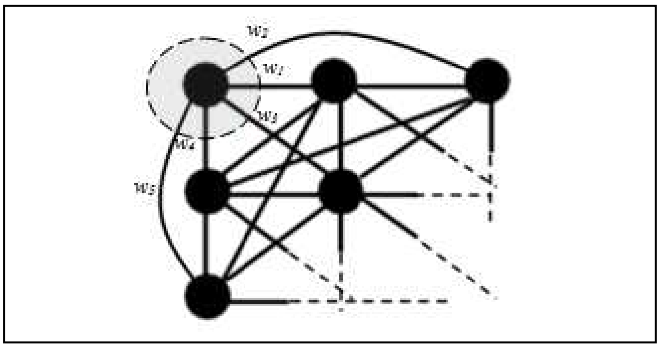


Fig. 1. Weight assigning for paired pixels.

context, the vertices are represented by the smallest element in the image, namely pixels [7]. Since each of these pixels holds colour information, grouping these pixels according their similarities and dissimilarities can lead to achieving the image segmentation goal. The edges, E, are set of elements consisting similarities and dissimilarities between pixels.

B. *Weight*

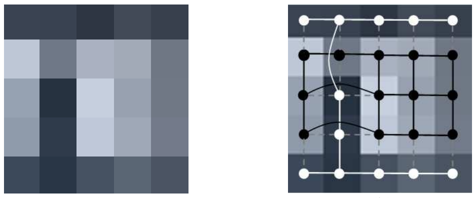
Prior to segmentation, initialising a weighted graph is required to construct the connectivity information of the pixels in an image. The edge set of E has each of them assigned with a weight, $w(i, j)$. Each $w(i, j)$ is a measurement of similarity between pixel i and pixel j. The value of $w(i, j)$ increases with the similarity degree between pixel i and pixel j. The degree of similarities between pixels decides important choices for grouping them into several segments. Fig. 1 illustrates an example of assigning weight for each of the paired pixels in a graph.

C. *Cuts*

Graph partitioning is done by cutting out edges with low value of weight. Weak weight of paired pixels indicates low similarity between the paired pixels. Hence, to partition the graph into two sub-graphs, the minimum cuts across edges are determined by finding the minimum value of

$$cut(A,B) = \sum_{i\in A, j\in B} w(i,j) \tag{1}$$

whereby A and B are sub-graphs with constraint of $A \cup B = V$, $A \neq \emptyset$, $B \neq \emptyset$, and $A \cap B = \emptyset$ [5, 7, 8]. A minimum cut criterion is introduced by Wu and Leahy such that it minimised the possible maximum cuts available across the sub-graphs. The criterion is a global optimal criterion for good image segmentation [8]. This criterion is implemented as image segmentation problem should be viewed by seeking all the possible solutions first. With the minimum cuts value determined, the edges with the weights indicated in (1) are cut out (removed) and two different segments (sub-graphs) are formed. This partition process continues recursively and stops until there are no further different segments exist. Fig. 2 shows the process of graph partitioning on an image with a size of 5 × 5 pixels.



(a) (b)

Fig. 2. Graph partitioning on an image with (a) the original image (b) the graph model, dashed lines denote weak similarity and non-dashed lines denotes strong similarities.

III. Normalised Cuts in Image Segmentation

Image segmentation with minimum cuts does give correct segmentation result but it favours in cutting out isolated pixels as shown in Fig. 3. Shi and Malik proposed an improved cuts algorithm named normalised cuts to alleviate the isolated pixels problem by introducing a disassociation measure for normalised cut, *Ncut* as shown in (2),

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)} \tag{2}$$

whereby $assoc(A, V) = \Sigma_{i\in A, j\in V}\ w(i, j)$ and $assoc(B, V) = \Sigma_{i\in B, j\in V}\ w(i,j)$. The proposed measure eliminates the occurrence of partition that cuts out isolated small set of pixels by formulating the cut as a fraction of the total edges that paired with all the pixels in the graph. The relation between disassociation and association can be described as in (3).

$$\begin{aligned} Ncut(A,B) &= \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)} \\ &= \frac{assoc(A,V) - assoc(A,A)}{assoc(A,V)} + \frac{assoc(B,V) - assoc(B,B)}{assoc(B,V)} \\ &= 2 - \left(\frac{assoc(A,A)}{assoc(A,V)} + \frac{assoc(B,B)}{assoc(B,V)} \right) \\ &= 2 - Nassoc(A,B) \end{aligned} \tag{3}$$

Based on Eq. 3, minimising the dissociation between partitions denotes maximising the association between partitions [5].

A. *Solving Normalised Cuts as Eigenvector Problem*

There are two main matrices to be formed, the weight matrix, **W** and the diagonal matrix, **D**. For an image with a size of $m \times n = N$ pixels, the **W** matrix would be in the size of $N \times N$. This goes the same for the **D** matrix that has the similar size with **W**. **W** is a similarity matrix where contains elements of $w(i, j)$ that denotes similarity between pixel i and pixel j. To relate **D** with **W**, the element of the diagonal in **D** is defined as following,

$$d(i) = \sum_j w(i,j) \tag{4}$$

The $d(i)$ denotes the sum of the weights that connect from pixel i to other pixels [1, 5]. With these two matrices, the minimum of the normalised cut is solved into the equation in the following,

$$\min Ncut = \min y \frac{y^T(\mathbf{D}-\mathbf{W})y}{y^T\mathbf{D}y} \tag{5}$$

whereby y is an vector with a size of N × 1 comprising elements with each of them as the attribute of each pixel in the image. Finally, (5) is then simplified into general eigenvalue problem.

$$(\mathbf{D}-\mathbf{W})y = \lambda\mathbf{D}y \tag{6}$$

with y as the eigenvector and λ as eigenvalue. Eigenvector corresponds to the second smallest eigenvalue is chosen for the image segmentation [5].

B. Simultaneous K-way Cut

Shi and Malik defined a simultaneous *k-way* cut as in

$$\begin{aligned} Ncut_k(A_1, A_2, ..., A_k) = & \frac{cut(A_1, V-A_1)}{assoc(A_1, V)} \\ & + \frac{cut(A_2, V-A_2)}{assoc(A_2, V)} \\ & + ... + \frac{cut(A_k, V-A_k)}{assoc(A_k, V)} \end{aligned} \tag{7}$$

This practically outputs of k segments in a single iteration [5, 7]. The solution to it is to use more than one eigenvector to add extra possible segments. There is proof showing that using more number of eigenvectors in the cut algorithm, the produced segmentation results will be finer [9].

C. Clustering on Eigenvector

As each element of the eigenvector gives a description value of the particular pixel within two partitions, k-means clustering is used to do the grouping on the eigenvector. In this paper, the weighted graph is partitioned based on pixels' two attributes, the colour and spatial location of them. Hence, each of the element in **W**, $w(i, j)$ is described as equation in the following,

$$w(i,j) = e^{\frac{-\|\mathbf{F}(i)-\mathbf{F}(j)\|_2^2}{\sigma_I}} * \begin{cases} e^{\frac{-\|\mathbf{X}(i)-\mathbf{X}(j)\|_2^2}{\sigma_S}} & , \ if \ -\|\mathbf{X}(i)-\mathbf{X}(j)\|_2^2 < r \\ 0 & , \ \text{otherwise} \end{cases} \tag{8}$$

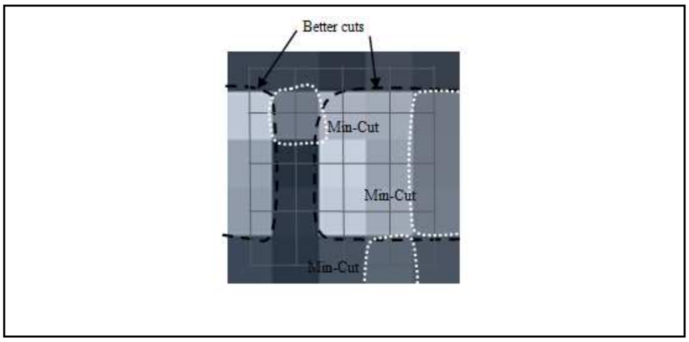


Fig. 3. Illustration of minimum cuts that cut isolated pixels

$\mathbf{F}(i)$ is feature vector based on intensity value in colour of pixel i and $\mathbf{X}(i)$ is the spatial location of the pixel. A threshold, r, which indicates the distance between pixel i and pixel j is set. When the distance of the paired pixels is more than r number of pixels apart, zero weight is set for the paired pixels. Image segmentation is then achieved by clustering on the eigenvector. Fig. 4 shows the segmentation process and the clustering process on the selected eigenvector. Most of the pixels lie near the centroid of cluster 2.

IV. Image Segmentation in Two Stages

Due to the normalised cuts criterion in which it globally views image as graph, every possible connected paired of pixels are considered. An image that has a size of $m \times n$ pixels will require the normalised cuts algorithm to construct a **W** matrix with a size of $(m \times n) \times (m \times n)$ pixels. Imagine that an image with a size of 360 × 270 pixels will end up computing 97200 × 97200 elements in **W** matrix, which is indeed a huge matrix for computer memory storage and also impractical to solve this giant size matrix. This will hamper the eigenvalue problem solving. Because of the issue mentioned above, most of the researchers would have the image to be resized into smaller number pixels during prior to image segmentation. However, not all images are practically to be shrunk. Figure 5 demonstrates images in different size.

A. Image Cells

To reduce the large memory usage, it is suggested that a high resolution image be divided into equal size of sub-images which we called them as image cells or sub-images [10, 11]. The first stage of image segmentation is begun with independent local segmentation in each of the image cells. Normalised cuts algorithm is then performed to segment out k_1 number of segments for every image cell. To save time on the local segmentation computation, a preliminary check is scanned across every image cell to determine whether the particular cell is necessary to have segmentation in it. In other words, no segmentation will be performed when all the pixels in the image cell is homogeneous.

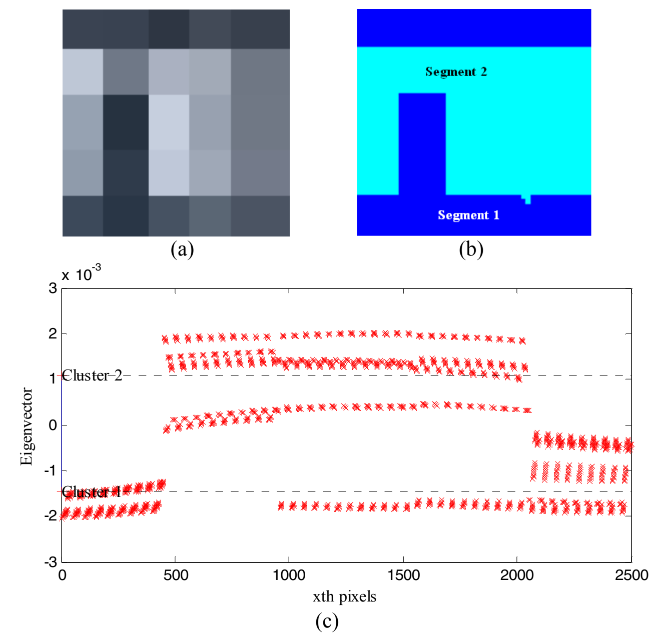


Fig. 4. Image segmentation using normalised cuts: (a) input with image size of 50 × 50 pixels, (b) output and (c) clustering process on eigenvector.

Hence, the image cell alone will be considered a segment. In this segmentation stage, over-segmentation is likely to happen as discrimination degree in an image cell is reduced as compared to whole image directly. Nevertheless, the over-segmentation will help to reduce the tendency of segmented object boundaries to be missed out and also producing finer segmentation. As mentioned previously, segments are produced from every image cells that have normalised cut on it before hand, using the simple k-means clustering. The segmentation done in a cell is independent with the segmentation done in other cell. Segmented clusters from the image cells are then used for second stage segmentation.

B. Segments Merging

Segments from every image cells are represented by nodes containing the median value of colour brightness of the pixels in the segments and centroid location of the segments. The computed nodes are then used for second stage segmentation using normalised cuts algorithm. The number of nodes is corresponded with the total number of segments are produced from the first stage segmentation. For example, 21 segments collected from across the image cells will have 21 nodes to represent the segments.

The computed nodes are again viewed as graph structure using the normalised cuts algorithm. Segments from the first stage image segmentation will be merged together when their corresponding nodes are sharing common similarity based on color and spatial location attributes. The second stage image segmentation is complete after final segments are produced. Fig. 6 shows a summary of the two-stage image segmentation.

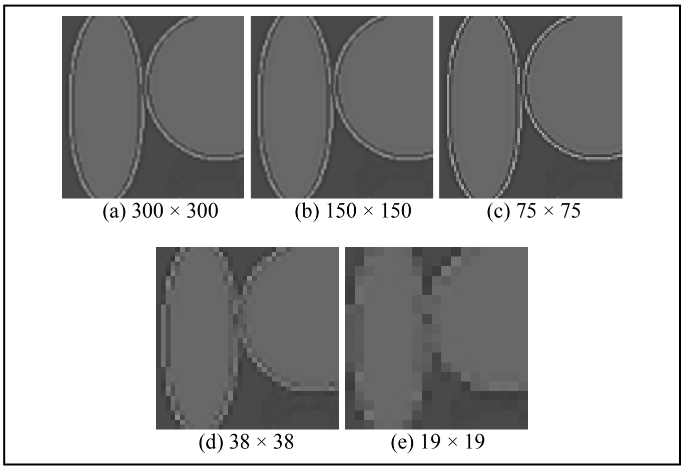


Fig. 5. Image with various sizes.

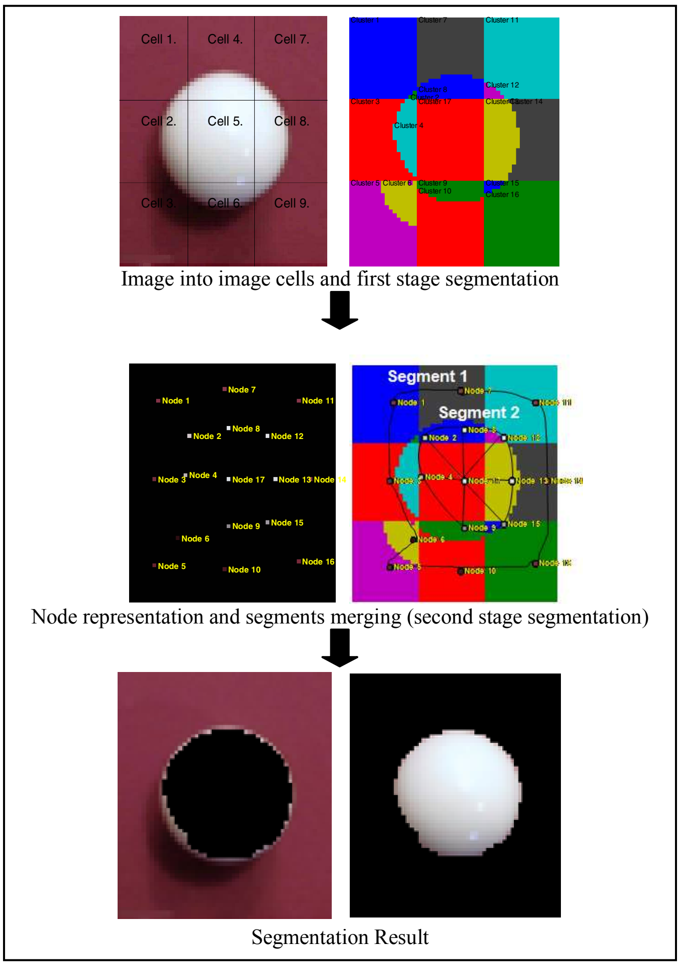


Fig. 6. Summary of two-stage image segmentation

V. EXPERIMENTAL RESULTS AND DISCUSSION

To illustrate the implementation of normalised cuts in image segmentation, synthetic and natural images taken from the Berkeley Segmentation Dataset (BSDS500) [12] are used.

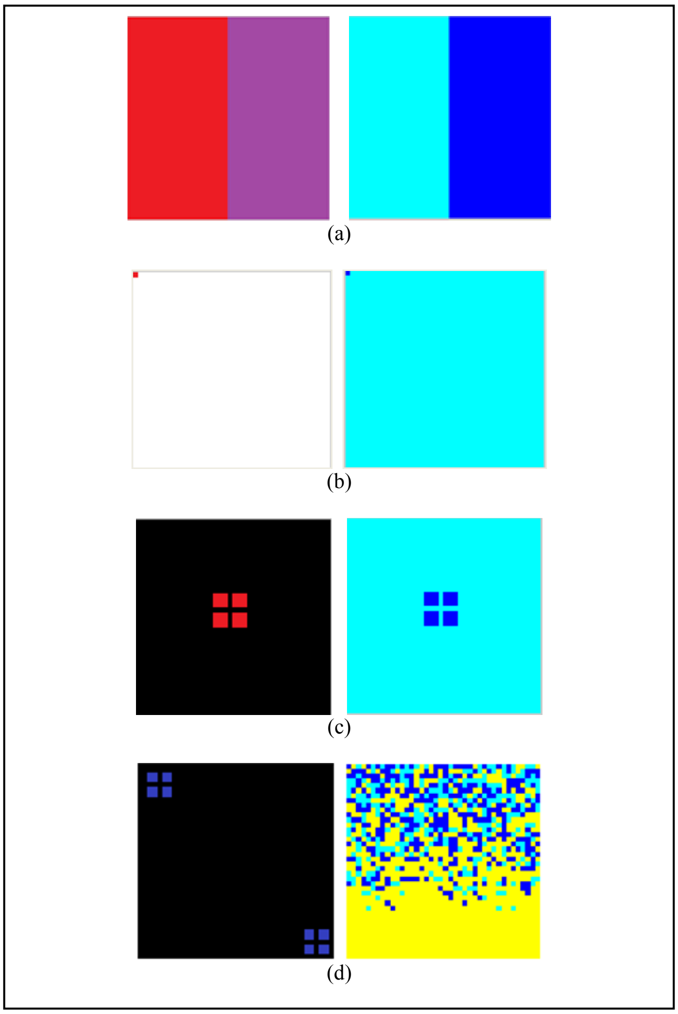


Fig. 7. Synthetic image segmentation with results on the right.

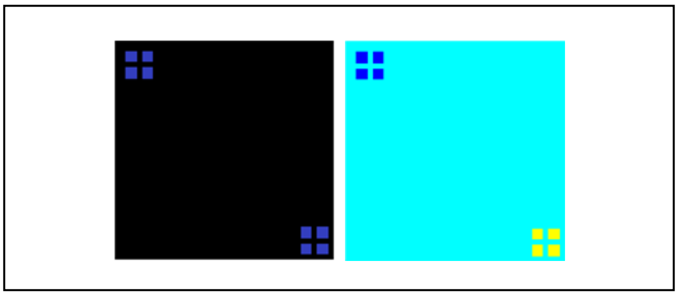


Fig. 8. Synthetic image segmentation based on colour and spatial location attributes.

A. *Synthetic Image Segmentation*

Synthetic image segmentation based on feature vector, $\mathbf{F}(i)$ only is carried out to show its grouping performance based on colour similarities only. The $\mathbf{F}(i)$ used here is based on *h, s, v* values from HSV colour space and is defined as following,

$$\mathbf{F}(i) = [v, v \cdot s \cdot \sin(h), v \cdot s \cdot \cos(h)](i) \tag{9}$$

The segmentation results of several synthetic images with the size of 40 × 40 pixels are shown in Fig. 7. It can be seen that

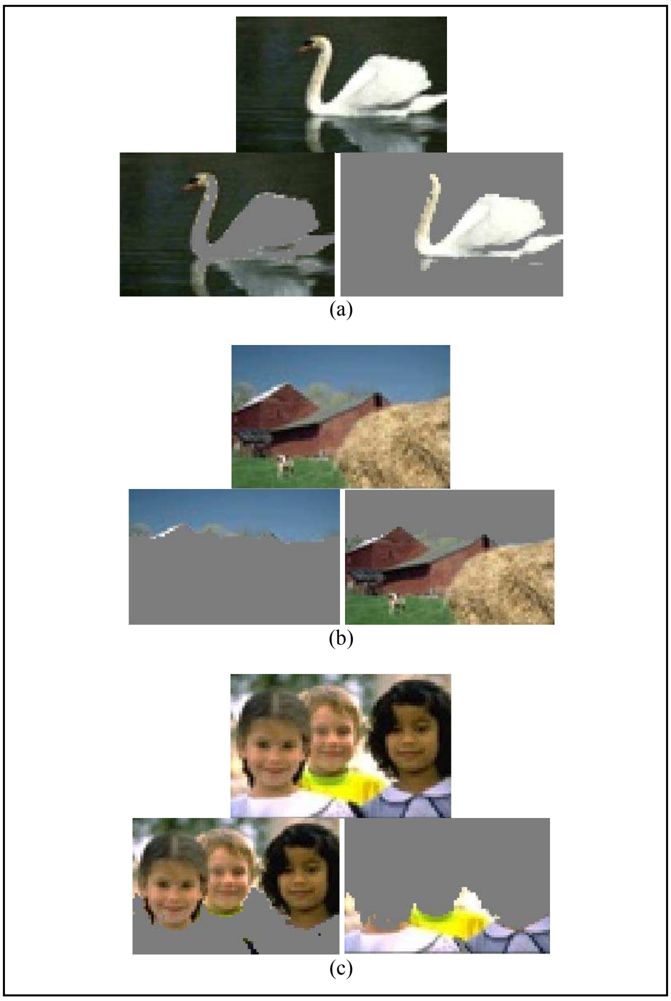


Fig. 9. Natural image segmentation.

image segmentation based on colour attribute only, incorrect segmentation result is produced when trying to produce 3 distinctive segments from it. Another important information should be included, that is spatial location information $\mathbf{X}(i)$. With the information included, correct segmentation result is produced as shown in Fig. 8. The $\mathbf{X}(i)$ is important to maximise the dissimilarities between pixels and to achieve a balanced segmentation. Since every pixel in the image are considered in graph partitioning, as in the second image in Fig. 7, despite only one distinctive pixel presents, the normalised cuts is still able to segment out the pixel. However, when there are several distinctive pixels for example with 6 different colours in it, the nature of normalised cut whereby it alleviates isolating pixels would be suppressed. With this case happened, optimising the clustering algorithm and selecting the suitable eigenvector will play as prominent roles in defining the correct segmentation.

B. *Natural Image Segmentation*

Natural image with more complex content increases the difficulties in grouping the pixels when using normalised cuts algorithm. Due to limited memory storage, the size of the natural images tested are only merely in 51 × 77 pixels. Based on the results presented in Fig. 9, the normalised cuts algorithm is able to segment the foreground and background objects but for the third image, part of the background and the children's faces are grouped as same segment. The absence of higher level knowledge or priori knowledge will never have good segmentation that near human perception. Artificial intelligence technique such as Artifical Neural Network which is normally used for recognition purpose [13], can be implemented to assist segmentation that can relate to human perception. The same images in the size of 321 × 481 are used in the two-stage segmentation as shown in Fig. 10. The results produced almost similar with the one done in single stage image segmentation except for the third image. The third children on the right are considered another segment which is complementary with the two children on the left. It is believed that image segmentation in two stages decreases the globally view capability which means the graph partitioning is favourer with local problem than the global problem. This poses a disadvantage to image that has sparse content in it although this approach reduces the computation cost.

VI. CONCLUSIONS

This paper presents a study on image segmentation using normalised cuts algorithm. The study leads more understanding on the robustness of normalised cut in terms of grouping and partitioning. Two-stage image segmentation can be implemented to help reducing unnecessary image segmentation in particular region instead of performing segmentation on whole image [10, 14]. Segmentation on image part by part individually also helps to speed up the computation time of similarity measurement in normalised cuts algorithm. It can be concluded that the normalised cuts algorithm requires some fine tuning tasks to produce correct segmentation. Soft clustering technique can be implemented for better segmentation result. This clustering algorithm allows optimal clustering on the eigenvector.

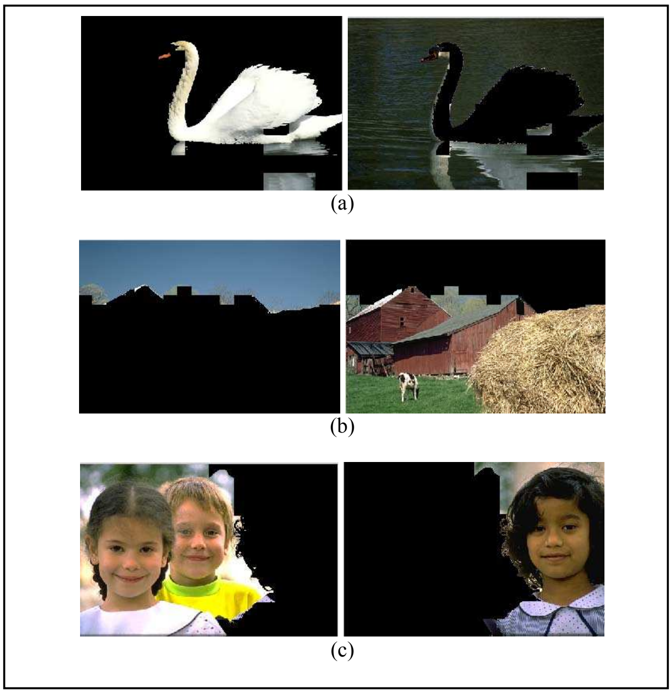


Fig. 10. Natural image segmentation in two stages.

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