# Enhancement of Particle Filter Resampling in Vehicle Tracking via Genetic Algorithm

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***Abstract* — Vehicle tracking is an essential approach that can help to improve the traffic surveillance or assist the road traffic control. Recently, the development of video surveillance infrastructure has incited the researchers to focus on the vehicle tracking by using video sensors. However, the amount of the on-road vehicle has been increased dramatically and hence the congestion of the traffic has made the occlusion scene become a challenge task for video sensor based tracking. Conventional particle filter will encounter tracking error during and after occlusion. Besides that, it also required more iteration to continuously track the vehicle after occlusion. Thus, particle filter with genetic operator resampling has been proposed as the tracking algorithm to faster converge and keep track on the target vehicle under various occlusion incidents. The experimental results show that enhancement of the particle filter with genetic algorithm manage to reduce the particle sample size.**

***Keywords - Vehicle tracking; Particle filter; Resampling; Genetic Algorithm***

## I. Introduction

Recently, the amount of the on-road vehicles has been increase apparently. Meanwhile, the incidents that created by the users of the vehicle are also elevated. Thus, vehicle tracking has drawn the attention among the researchers due to its numerous fields of applications such as traffic surveillance and security monitoring system, advance driver assistant system (ADAS), road traffic control assistant system and navigation system [1]. There is various types of sensors have been implemented in the vehicle tracking. In this paper, the video sensor has been chosen as the media used for processing due to the development of the video surveillance infrastructure has been growth vastly in recent years. Moreover, video sensors also can provide a wide range of information that used to describe the vehicle. For instance, the features vehicle such as colour, shape, edge and motion can be obtained by extract the data from the video sensor via image processing techniques.

Besides that, due to the high congestion of the traffic flow [2], the occlusion and overlapping will become the common scenario. Furthermore, occlusion and overlapping between vehicles is a challenging task in surveillance system via video sensor. Thus, the complexity and difficulties cause by the occlusion problems has become the driving force to the researchers to study and develop an effective and efficient vehicle tracking algorithm.

Vehicle tracking could lead to non-linear and non-Gaussian situations due to the dynamic changes of the vehicle flow. Thus, particle filter has been chosen in this research due to its ability to deal with non-linear and non-Gaussian situations. Although, particle filter has the ability to overcome the dynamic changes problems. However, particle filter will undergo particle degeneracy after a few iteration of processing. Nevertheless, particle degeneracy problem can be solved by implement a huge amount of particles however it is always impractical due to the computational complexity. Thus, an efficient and effective resampling approach will be required to solve the particle degeneracy problem. Therefore, an enhancement of the particle filter resampling algorithm will be implemented to track the target vehicle under overlapping situations.

## II. Reviews of Object Tracking

Throughout the literature, there are many different algorithms have been developed for object tracking purpose. For instance, the techniques such as Kalman filter, Markov Chain Monte Carlo, optical flow and particle filter are the well known object tracking techniques. Although there are many type of object tracking algorithm, each of the techniques have the pros and cons. For example, Kalman filter is a framework for estimating the object state and using the measurements to update the state estimation. Hence, Kalman filter is act an estimator that predicts and corrects the states of linear process [3]. To solve the non-linear case, the extended version of Kalman filter can be implemented to change the measurement relation of the current estimate become linear [4]. However, when the nonlinearity is inaccurately approximated by the algorithm, the estimated results will be diverged and hence lead to an inaccurate tracking result.

Moreover, Markov Chain Monte Carlo is one of the techniques that implemented for vehicle tracking purpose. However, the sample size implemented in the algorithm was an issue because a non-optimal sample size will affect the tracking accuracy [5]. Although the algorithm was able to track the overlapped vehicle with adaptive sample size but

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the algorithm still unstable and have some tracking result errors.

Besides that, optical flow also is a well known technique that used for object tracking. In research [6], the optical flow was used to detect and track the moving target. However, the optical flow was reflects to the motion field of the captured image. Thus, when there was overlapping occurred, the optical flow could hardly locate the target object or mislead by the obstacles.

Hence, particle filter has been chosen as the tracking algorithm in this study because it is a promising technique that can deal with non-linear situations [7, 8]. In research [9], the conventional particle filter will faced the particle degeneracy during the tracking process. The occurrence of particle degeneracy is because the low weight particles were selected after a few iterations and hence it blocks the further improvement of the algorithm.

In research [10], it states that the particle degeneracy can be solved by implement a huge amount of particles in the algorithm or by resampling the particles. Moreover, implement huge amount of particles is always impractical due to the high computational. Thus, resampling the particles becomes the suitable solution to deal with the degeneracy problem [11].

Besides that, the colour feature was implemented in research [10, 12] to track the vehicle and non-rigid objects. From the results shown, the algorithm with colour feature can accurately track the vehicle because colour feature was strong to deal with partial occlusion, scale invariants and rotation occurrence. However, the colour feature will have the limitation when the background is cluttered or the colour of the background similar with the target. Furthermore, in research [13] have been shows that the tracking algorithm with multiple features will provide a more accurate tracking results. Thus, in this study a multiple features of particle filter with genetic operator resampling algorithm has been proposed. The experimental results show that with this genetic operator resampling algorithm, the target object can be tracked with more accurate under various occlusion incidents.

## III. Particle Filter Framework

Particle filter also known as sequential Monte Carlo is a mainstream tracking technique to represent the propagation conditional density distributions when the observation probability density distributions involved in the process are non-linear and non-Gaussian. Moreover, particle filter algorithm is developed based on approximates the current state of the target vehicle by using previous observations state. In visual tracking, the observation state of the target is normally referred to the colour, edge, shape, texture and etc. which can characterize the target object. In this study, the colour feature and the shape feature has been selected as the features to describe the target vehicle model.

In general, particle filter approach is functioning based on three important stages which are prediction stage, measurement stage and follow by resampling stage. In the prediction stage, a set of particles which represent the state transition of the vehicle model will be generated. Moreover, the measurement stage is the stage that computes the weight for the particles based on the likelihood measurement. In addition, resampling stage is to avoid the particle degeneracy problems occur.

In this study, particle filter was developed to track vehicle in dynamic changes. Hence, the posterior probability density function $p(X_t \mid Z_t)$ and the observation probability density function $p(Z_t \mid X_t)$ computed in particle filter algorithm are often non-Gaussian. From the posterior probability density function, the state vector $X_t$ denotes state space of the tracked vehicle. Meanwhile, $Z_t$ denotes all the estimations state space.

As stated earlier, the main idea of particle filter is to approximate the posterior distribution base on a finite set of random weighted samples or known as particles $N_p$ . In addition, each weighted particles are drawn to represent the state estimation of the target vehicle according to the posterior distribution as shown in (1) where $x_t^i$ denotes the state of the target vehicle and $w_t^i$ denotes the weight that associate to the particle. Since $w_t^i$ is the weight that assign to each particle hence the limit for each weight of the particle is $w_t^i \in [0,1]$ . The whole set of particles weight should able normalized and sum up to one as shown in (2).

$$S_t^i = \{X_t^i, W_t^i\}_{i=1,2,3,\dots,N_p} \quad (1)$$

$$\sum_{i=1}^{N_p} w_i = 1 \quad (2)$$

### A. Prediction Stage

Prediction stage is the primary stage that initiates sample particles. Each particle is represents the estimated posterior position individually. However, with the implement a large amount of sample particles, the accuracy to estimate the state of the target vehicle will be increase. Unfortunately, by implement the large amount of the sample particles for estimation process, the computational cost will be highly expensive and vice versa for the least amount of particles implement. In the prediction stage, the prior probability density function can be obtained through (3). Based on the prior probability density function obtained, the posterior probability density function can be computed through the updated stage by using the Bayers' rule as shown in (4).

$$p(X_t \mid Z_{1:t-1}) = \int p(X_t \mid X_{t-1}) p(X_{t-1} \mid Z_{1:t-1}) dX_{t-1} \quad (3)$$

$$p(X_t \mid Z_{1:t}) = \frac{p(Z_t \mid X_t)p(X_t \mid Z_{1:t-1})}{p(Z_t \mid Z_{1:t-1})} \tag{4}$$

### B. *Measurement Stage*

In measurement stage, the weight of each particle is computed based on the likelihood probability from the features of target vehicle. Hence, the observation state of the target vehicle can be colour, shape, edge or texture that extract from the model of the target vehicle. When the features of the target vehicle have been extracted, the measurement likelihood needs to compute accordingly. In this study, the weight of the particles will be computed based on shape and colour features likelihood. It uses to compute the weight of the particles based on the similarity histogram of the target vehicle and the reference vehicle model. The colour likelihood is computed using equation in (5) and the shape likelihood is computed using equation (6) where $\sigma$ in (5) and (6) is the adjustable standard deviation which can be chosen experimentally.

$$.\varphi_c = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{b_{dist}{}^2}{2\sigma^2}} \tag{5}$$

$$\varphi_s = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{H_{dist}{}^2}{2\sigma^2}} \tag{6}$$

The colour likelihood is determined by using Bhattacharyya distance, $b_{dist}$ [14] whereas shape likelihood is determined by using Hausdorff distance, $H_{dist}$ [15]. The value of distance will become smaller if the target vehicle is similar with the reference vehicle and vice versa. Both of the likelihood will combine together to compute the weight of the particles as shown in (7) where α is the weight constant.

$$w_t^i = \alpha(\varphi_c) + (1-\alpha)(\varphi_s) \tag{7}$$

After likelihood is computed, the weight for the particle will be updating as shown in (8) where $q(X_t \mid X_{t-1}, Z_t)$ is the proposal distribution.

$$w_t^i = w_{t-1}^i \times \frac{p(Z_t \mid X_t)p(X_t \mid X_{t-1})}{q(X_t \mid X_{t-1}, Z_t)} \tag{8}$$

After the weight for the particle is updated, the weight of the particles will undergo weight normalization as shown in (9) before the predictive posterior density function is approximated. In the particle filter algorithm, the posterior probability density function computed from the prior density function is represented by a set of weighted particles. Furthermore, the weight of the particles is computed in discrete nature. Hence, the posterior density function can be obtained through (10).

$$W_t^i = \frac{w_t^i}{\sum_{i=1}^{N_p} w_t^i} \tag{9}$$

$$p(X_t \mid Z_{1:t}) \approx \sum_{i=1}^{N_p} W_t^i \delta(X_t - X_t(i)) \tag{10}$$

When the predictive posterior distribution for each particle is obtained, the particle filter algorithm will enter the final step. In the final step, the position of the target vehicle will be estimated by taking the mean of the predicted state. For instance, the mean state of the target vehicle can be calculated by using (11) where $S_t^i$ is shown in (1).

$$E(X_t) = \frac{1}{N_p} \sum_{i=1}^{N_p} S_t^i \tag{11}$$

### C. *Resampling Stage*

Although the position of the target vehicle can be predicted without the resampling stage, however the result obtained is not convincing after a few iteration. This is because particle filter will face an inherent problem which is particle degeneracy. When particle degeneracy problem occur, one particle will experience negligible weight because the variance of the important weight will increase over time. The particle degeneracy is denotes that a huge computation effort is needed to update particles whose the weight contribute to the state approximation is almost zero. Hence, the particle degeneracy problem is cannot avoid but it can be solved by resampling or implement a huge amount of particles.

Implement a huge amount of particles is always impractical due to the high computational. Hence, resampling is the best solution to overcome the particle degeneracy problem. Resampling stage should apply at the beginning of iteration, by eliminating those low weight particles and only concentrating on those high weight particles. The resampling is normally applied by replacement basis. This means that a new set of particles will be defined to replace those eliminated particles.

In order to determine the occurrence of the particle degeneracy problem, the effective sample sizes need to be computed by using (12). From (12), the weight $w_t^i$ is referred as the true weight as shown in (13). Nevertheless, the true weight is very hard to compute exactly. Thus, an estimate of effective sample sizes can be obtained through (14) where $w_t^i$ is the normalised weight.

$$N_{eff} = \frac{N_p}{1 + Var(w_t^{*i})} \tag{12}$$

$$w_t^{*i} = \frac{p(x_t^i \mid z_{1:t})}{q(x_t^i \mid x_{t-1}^i, z_t)} \tag{13}$$

$$\hat{N}_{eff} = \frac{1}{\sum_{i=1}^{N_s} (w_t^i)^2} \tag{14}$$

Notice that if $\hat{N}_{eff}$ is small or $\hat{N}_{eff} < N_{thres}$, which means the particle degeneracy problem is occurred and resampling is required. Hence, the particle filter will be recursively repeating the prediction stage and measurement stage until the stopping criteria was fulfil. As a result, resampling is taking an important role in the particle filter algorithm in order to obtain an accurate tracking result.

## IV. Proposed Genetic Operator Resampling

The conventional resampling step can reduce the effect of particle degeneracy but unfortunately it will create another practical problem. The problem created was known as sample impoverishment. Sample impoverishment will occurred when the particles with heavy weight are statistically selected many times. Thus, the estimated posterior state will contain many repeated locations and lead to loss of diversity among the particles. After a few iterations, all the particles will collapse to a single position and hence the algorithm will unable to continuously track the target vehicle. In order to avoid the particles diversity, genetic operator will implement in the particle filter resampling algorithm.

In order to generate a good quality children solution, the selection of the parents for crossover was became an important step. Selection is used to improve the quality of the population by giving individuals of higher quality to generate new offspring or known as children. By implement rank selection, the particles will be given a rank according to the weight of the likelihood computed. The most heavy particles will assign with higher rank while the lighter weight particles will assign with lower rank. After all the particles have been ranked, the algorithm will be randomly select two particles as the first parent and the second parent. Since the heavy weight particle has been assigned with higher rank, and hence the chances for the heavy weight particle being selected as the parent will be higher. In this research, the parent was represents the position of the target vehicle.

After performed the rank selection, the next step will be crossover process. In this study, arithmetic crossover will be implemented in the particle filter resampling algorithm. The advantage of arithmetic crossover was it always produces feasible children by containing both parents characteristic. The children generated by using arithmetic crossover were shown in (15) and (16).

$$C1 = P1 \times \alpha + P2 \times (1 - \alpha) \tag{15}$$

$$C2 = P2 \times \alpha + P1 \times (1 - \alpha) \tag{16}$$

where α is a weight factor with a limit of zero to one, *P1* and *P2* was the parent solutions and *C1* and *C2* are the children solutions. The weight factor is used to determine the fraction of characteristic from parent solutions contribute to the children solutions. In this study, the weight factor was set as 0.7 which means the first children will preserve most of the first parent characteristic meanwhile second children solution will preserve most of the second parent characteristic. By applying arithmetic crossover in the particle filter resampling algorithm, the low weight particles will be eliminated first meanwhile the heavy weight particles will be preserved. Hence, arithmetic crossover operator will generate the children solution to replace those eliminated low weight particles. By using arithmetic crossover the estimated position for the target vehicle will be converge to the real position. Hence, a more accurate tracking result can be computed.

After the genetic crossover process, the children solutions will undergo mutation process. Mutation operator is a process to maintain the genetic diversity from one generation to the next generation. Besides that, the mutation operator also acts as a final checking state to recover the good information which might be lost during selection and crossover stages. Mutation was played an important part in the genetic algorithm to prevent the population stagnating at the optimal position. Mutation occurs during evolution according to the mutation rate defined by the user. Furthermore, the mutation rate needs to be set fairly low. In this research, the mutation rate was set as 1 percent in order to avoid the loss of fit solutions and affect the convergence of solutions. If the mutation rate was hit, a new children will be generated with the position estimated add with a random number with the limit of zero to one. The development of genetic operator in particle filter resampling algorithm will illustrate in Table I.

## V. Result and Discussion

In this section, the result of vehicle tracking using conventional resampling (Fig. 1) will be compared to the result of vehicle tracking using a genetic operator resampling algorithm (Fig. 2). In both cases, the particle size was initialized as 200 particles. As shown in Fig. 1 and Fig. 2, the cross symbol is represents the estimated position of the vehicle. Meanwhile the red colour solid boundary box is indicates the location of the target vehicle. The target localization was determined by the mean value of the estimated position of the particle.

Referring to Fig. 1 and Fig. 2, the tracking can categorize into four situations, such as before occlusion, partially occlusion, fully occlusion and after occlusion. From the sequence of results shown in Fig. 1 and Fig. 2, the genetic operator resampling provide a more promising and accurate tracking result.

TABLE I. PROPOSED GENETIC OPERATOR RESAMPLING ALGORITHM

**MEASUREMENT & WEIGHT UPDATE:**
Compute the weight of the particle
Summation of particle weight
Normalize the weight
Calculate $\hat{N}_{eff}$

$$\hat{N}_{eff} = \begin{cases} < N_{thres} & \text{Resampling} \\ \geq N_{thres} & \text{Acceptance} \end{cases}$$

**RESAMPLING:**
Performed Rank Selection
Arithmetic Crossover
Generate mutation rate
**IF** mutation < 1%
$X_t^i = X_t^i + rand(0,1)$
**ELSE**
$X_t^i = X_t^i$
**END IF**
**LOCALIZATION:**
$(x, y) = E(X_t)$

In case 1, the target vehicle was free of occlusion as shown at Frame 5 in Fig. 1 and Fig. 2.The results obtained shows that the target vehicle was accurately being tracked by the conventional and genetic operator resampling algorithm. This is because before occlusion, the information used to describe the target vehicle was clear and without influence by the obstacles.

In case 2, the target vehicle was partially occluded by another moving vehicle as shown at Frame 21 in Fig. 1 and Fig. 2. From the results shown, the conventional resampling was merely tracks the target vehicle. This is due to the information of the target vehicle has been influenced by the moving vehicle. However, the target vehicle still able being tracked by using genetic operator resampling although there are appearance of another vehicle.

In case 3, the target vehicle was fully occluded by the moving vehicle as illustrated at Frame 33 in Fig. 1 and Fig. 2. From the results obtained, the conventional resampling was unable to estimate the location of the target vehicle because the information of the target vehicle has been lost. Meanwhile, the genetic operator resampling was able to locate the target vehicle.

In case 4, the target vehicle was after occluded by the moving vehicle. At Frame 41 and Frame 49 in Fig. 1 and Fig. 2, the target vehicle was reappeared after occlusion. The conventional resampling algorithm was merely track the target vehicle. However, the genetic operator resampling was accurately resume the tracking process. The information of the target vehicle was influenced by the moving vehicle. Hence, the conventional resampling will require more time to gain back the information of the target vehicle. However,

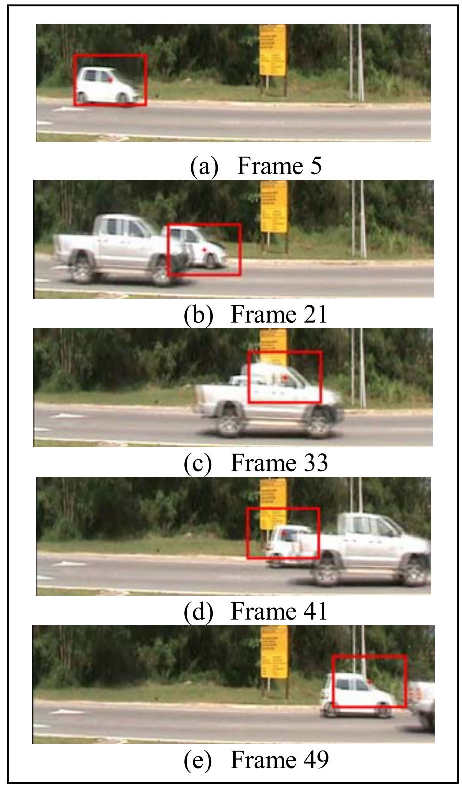


Figure 1. Result of vehicle tracking by using conventional resampling.

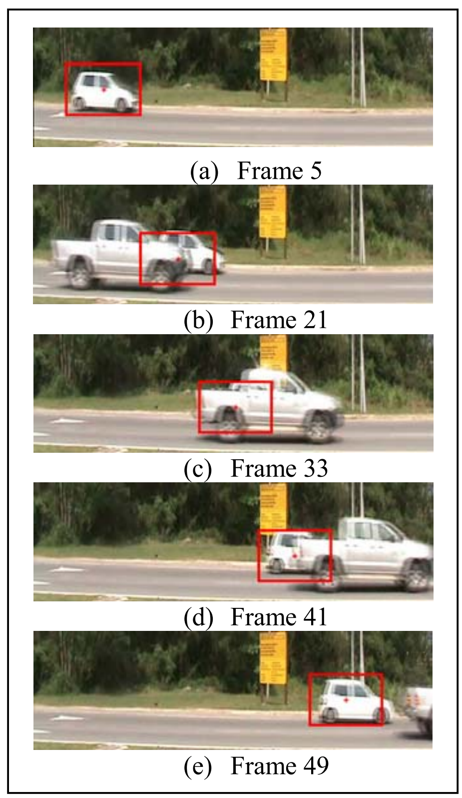


Figure 2. Result of vehicle tracking by using proposed genetic operator resampling algorithm.

the genetic operator resampling was able recover the information in short duration.

The RMSE of the conventional resampling and the developed genetic operator resampling is plotted as shown in Fig. 3. From the results obtained, it was clearly shows that the RMSE for the genetic operator resampling was lower than the RMSE for the conventional resampling. Hence, it can conclude that the improved resampling was giving a higher accuracy in tracking vehicle under occlusion situations.

## VI. Conclusion

As mentioned earlier, the accuracy of the particle filter could diminish by particle degeneracy. Thus, resampling was played an important role in the algorithm. Although, the conventional resampling able to track the target vehicle when before occlusion. Unfortunately, it was unable to continuously and accurately track the target vehicle after occlusion. Thus, the implementation of the genetic operator resampling algorithm is capable to alleviate the tracking difficulties under various occlusion situations. From the results shown, the performance and robustness of the proposed resampling algorithm was promising and tested under different tracking conditions. Hence, it can conclude that the proposed resampling algorithm has been improved the accuracy of the tracking results.


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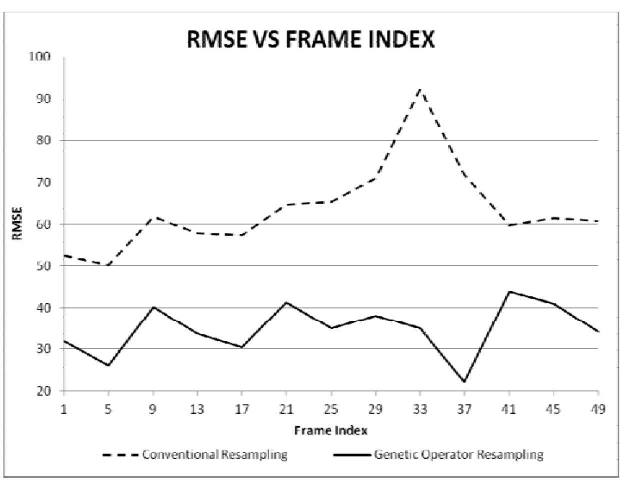


Figure 3. Graph of RMSE vs frame index for conventional resampling and genetic operator resampling.

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